Math Solutions
PROFESSIONAL DEVELOPMENT

Exploring Halves

A Lesson with Third Graders

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In this lesson, students explore halves, looking for patterns between numerators and denominators. Maryann Wickett created this simple yet powerful fractions lesson and then built on it, doing an activity from Marilyn Burns's Teaching Arithmetic: Lessons for Introducing Fractions, Grades 4–5 *(Math Solutions Publications, 2001).*

My-third grade students had experience using fraction kits to informally explore equivalent fractions (for example, four-sixths and two-thirds). They clearly understood what numerators and denominators represent when working with common fractions. It was time to push their thinking a bit. I wanted to give them an experience that would encourage them to look for patterns between the numerators and denominators of common fractions, in this case, halves. My objective was for them to discover that when any numerator is exactly one-half the denominator, the fraction is equivalent to one-half.

Before class I wrote on the board:

$$\frac{1}{2} \quad \frac{2}{4} \quad \frac{3}{6} \quad \frac{4}{8} \quad \frac{5}{10} \quad \frac{6}{12} \quad \frac{7}{14} \quad \frac{8}{16}$$

What pattern do you notice between the numerator and the denominator of each of these fractions?

As the students came in from recess, I asked them to read and then think quietly for a few moments about the prompt on the board. When all the students had a chance to settle and consider what I'd written, I asked them to take out their math journals, copy the prompt, and record their thoughts. As they worked, I circulated among the students, reading what they wrote. I realized that several students were not clear on what I was asking. They were noticing patterns among the numerators as one group and among the denominators as another; for example, they wrote that the numerators increased by one and the denominators increased by two. I asked for the students' attention and stated that the patterns some students had noticed were indeed true of this set of fractions, but what I was asking them to do was to look at the numerator and denominator of each fraction to find a relationship or pattern. "For example, how is the numerator of one related to the denominator of two? How is the numerator of two related to the denominator of four?" I asked. As expected, I heard several ohs and ahs as the students then understood what I was asking. Students returned to their writing, and when they had finished, I asked for their attention.

To begin a class discussion, I asked for a volunteer to share what he or she discovered. Hands leaped into the air. I called on Stephanie, who said, "The numerator is half of the denominator." I asked Stephanie to give an example. She said, "Well, one is half of two. One is the numerator and two is the denominator, so the numerator is half the denominator. It works for all of them. Eight is half of sixteen." I recorded Stephanie's idea on the board and, as I always do, I asked if she agreed with what I had written and I asked for her permission to write her name next to what I'd written. I called on Jared next.

Jared said, "The numerator multiplied by two equals the denominator. If you multiply the numerator of one by two, you get two, which is the denominator. It works for all of them. For three-sixths, if you multiply three times two, it equals six." I recorded Jared's idea under Stephanie's.

Lilly shared next. She said, "I think my idea is like Jared's but different. I noticed that the numerator plus the numerator equals the denominator. I think it is sort of like Jared's because adding a number to itself is like multiplying by two, except it's adding." I was impressed that Lilly had connected her idea to Jared's and had recognized that doubling a number results in the same total as multiplying a number by two. I recorded Lilly's idea on the board.

Several children still wanted to share. I allowed all who wanted to do so to share. The recording on the board looked as follows:

Stephanie: The numerator is half the denominator.

Jared: The numerator multiplied by two equals the denominator.

Lilly: The numerator plus the numerator equals the denominator.

Payal: The denominator divided by two equals the numerator.

Brandon: The denominator minus the numerator equals the numerator.

Tennisen: Double the numerator equals the denominator.

Garrett: All the fractions equal $\frac{1}{2}$.

After taking a moment to read the statements and consider them, Jordan started to wave her hand excitedly in the air. She said, "I noticed something. I think all of those things are just different ways of saying the same thing. Multiplying by two is the same as doubling, which is what Lilly said, so her idea and Jared's are the same. And dividing the denominator by two and one-half the denominator are the same thing. So Stephanie and Payal are saying the same thing in two ways." As she spoke, several other hands went up.

Katrina said, "I think Tennisen's idea is like Lilly's and Jared's. Tennisen said to double the numerator, which is like adding the numerator plus the numerator, which is Lilly's idea, and doubling is like multiplying by two, which is Jared's idea. So I think that Tennisen, Lilly, and Jared are saying the same thing in different ways." Many students nodded, indicating their agreement with Katrina.

Tennisen raised his hand tentatively. He said, "I am not sure all those ideas are the same. I don't think Brandon's is correct. Brandon said that the denominator minus the numerator equals the numerator." After a few moments of quiet think time to consider what Tennisen said, I asked the students to share their ideas with their table groups. The discussions were lively and some students were still uncertain. I called for their attention.

To help students gain some clarity about Brandon's statement, I used the fraction two-fourths as an example. Using magnetic fraction strips, I placed four fourths on the board. I reminded the students that Brandon's idea stated that the denominator minus the numerator would equal the numerator in each of the fractions on the board. I said as I removed two fourths, "The denominator of two-fourths is four. I have four fourths here. Brandon's idea states that if I subtract the numerator of two from the denominator of four, I will have the numerator of two. I am removing two fourths from the four fourths, and sure enough, I have two fourths left." Several ohs and ahs were uttered. I continued, "It seems to work for the fraction two-fourths. Let's see if it works for the fraction one-half." I repeated this process again for one-half and three-sixths. Then I asked the students to use their fraction kits to try it for themselves with the fractions four-eighths and eight-sixteenths. They were convinced and agreed that Brandon's idea worked for this set of fractions.

Tepanka said, "Actually, I think Brandon's idea is a different way of saying Payal's idea because subtracting the same number twice is like dividing by two, right?" I nodded my agreement.

To check Garrett's idea stating that all the fractions were equal to one-half, we used the fraction kit to test some of the fractions to see if they were indeed equivalent to one-half. They were.

From here, we moved into another lesson, *Fractions in Context,* which appears in Marilyn Burns's *Teaching Arithmetic: Lessons for Introducing Fractions.* The lesson presents a series of situations involving fractions. Students are asked if the situation can be represented by exactly one-half, about one-half, or more or less than one-half. Students easily used what they had discovered earlier to make sense of each of the situations presented. Typically, students doubled the numerator to see if it was equal to the denominator. If it was, then the situation could be represented by exactly one-half. If the numerator doubled was less than the denominator, they realized that the fraction representing the situation was less than one-half. If the numerator doubled was greater than the denominator, the fraction representing the situation was greater than one-half. Students soon realized that if the numerator doubled was close, but not exactly equal, to the denominator, then the fraction was close to one-half. Powerful understanding and thinking about fractions and number sense!